

地球流体乱流の数値解析

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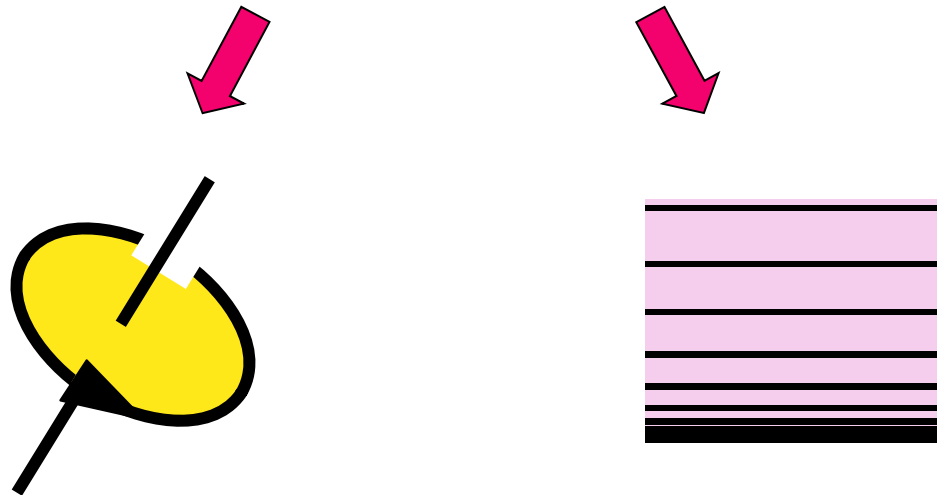
地球流体力学の特徴

舞台の特徴

薄い球殻内の運動であること
(2次元性/3次元性)

力学の特徴

回転と**密度成層**が異方性をもたらす



Navier-Stokes equation with the Boussinesq approximation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \theta \hat{\mathbf{z}}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla) \theta = \kappa \nabla^2 \theta - N^2 w$$

$$\nabla \cdot \mathbf{u} = 0$$

調和振動子の
のカップリング

where

$\mathbf{u} = (u, v, w)$: velocity

θ : temperature fluctuations

$N^2 = \frac{g\alpha}{T_0} \frac{\partial \bar{T}}{\partial z}$: Brunt - Väisälä frequency

成層乱流における数学的な問題

- ◆ 外力がない減衰乱流における全運動エネルギー減衰率

$$K = \frac{1}{2} \int u(t)^2 dV \sim t^{-\alpha}$$

- ◆ 外力がある定常乱流におけるエネルギースペクトル

$$E(k) = \frac{1}{2} \langle |\tilde{\mathbf{u}}|^2 \rangle(k) \sim k^{-\alpha}$$

- ◆ 成層乱流中を運動する粒子の分散

$$\frac{d\mathbf{x}_i}{dt} = u(\mathbf{x}_i(t)) \longrightarrow \langle |\mathbf{x}(t) - \mathbf{x}(0)|^2 \rangle \sim t^\beta$$

Lagrangian particle single particle dispersion

本研究の目的、内容

- ◆ 定常な成層乱流におけるエネルギースペクトルの特徴を大規模数値（格子点数 $\sim 2048^3$ ）により解析し、その生成メカニズムを渦運動の観点から考察すること

Numerical Methods

- ◆ forced simulations
- ◆ 2π -periodic box with $1024^3 \sim 2048^3$ grid points ($R_\lambda \sim 400$)
- ◆ 3rd order time-marching scheme
- ◆ Initial energy spectrum : $E(k) = 0$
- ◆ Force horizontal velocity components
- ◆ Add red noise to modes within a wave number band
($k_f \sim 5$)

Solving Ornstein-Uhlenbeck process (2nd order stochastic ODEs)

Characteristics of stratified turbulence

- ◆ Composite of “waves” and “turbulence”



“*Craya-Herring decomposition*” to separate waves and turbulence

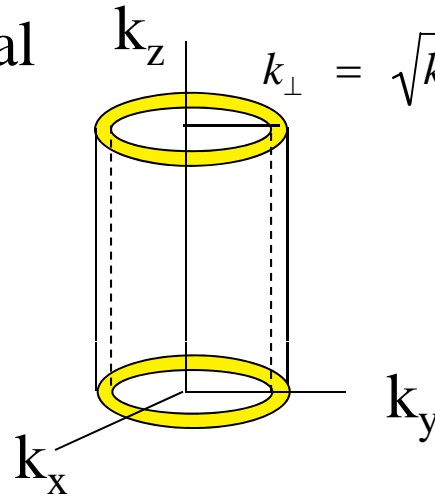
- ◆ Highly anisotropic



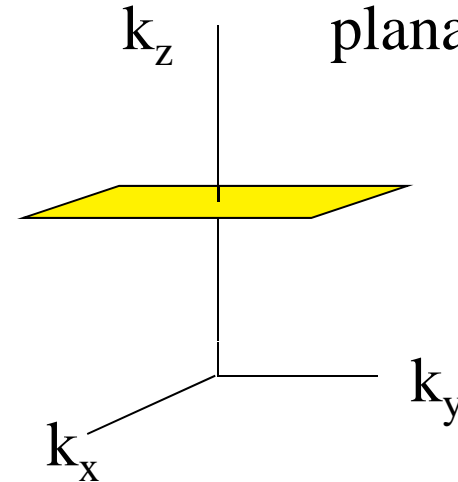
Need suitable averaging

$$\mathbf{k} = k_{\perp} \otimes k_z$$

cylindrical



planar



“Craya-Herring” decomposition

$$\nabla \cdot \mathbf{u} = 0$$

incompressibility



$$\mathbf{k} \cdot \tilde{\mathbf{u}} = 0$$

$\tilde{\mathbf{u}}$ is spanned by two independent vectors perpendicular to \mathbf{k}

$$\mathbf{e}_1(\mathbf{k}) = \frac{\mathbf{k} \times \hat{\mathbf{z}}}{\|\mathbf{k} \times \hat{\mathbf{z}}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2}} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}$$

$$\mathbf{e}_2(\mathbf{k}) = \frac{\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{z}}}{\|\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{z}}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2 + k_z^2} \sqrt{k_x^2 + k_y^2}} \begin{pmatrix} k_z k_x \\ k_z k_y \\ -(k_x^2 + k_y^2) \end{pmatrix}$$

$$\mathbf{e}_3(\mathbf{k}) = \frac{\mathbf{k}}{\|\mathbf{k}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2 + k_z^2}} \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}$$

orthnormal coordinates

$$\tilde{\mathbf{u}}(\mathbf{k}) = \phi_1 \mathbf{e}_1(\mathbf{k}) + \phi_2 \mathbf{e}_2(\mathbf{k})$$

$$\phi_1 = \tilde{\mathbf{u}}(\mathbf{k}) \cdot \mathbf{e}_1(\mathbf{k})$$

$$= \frac{1}{\sqrt{k_x^2 + k_y^2}} (k_y \tilde{u} - k_x \tilde{v})$$

$$= \frac{i}{\sqrt{k_x^2 + k_y^2}} \tilde{\omega}$$

(vortical)

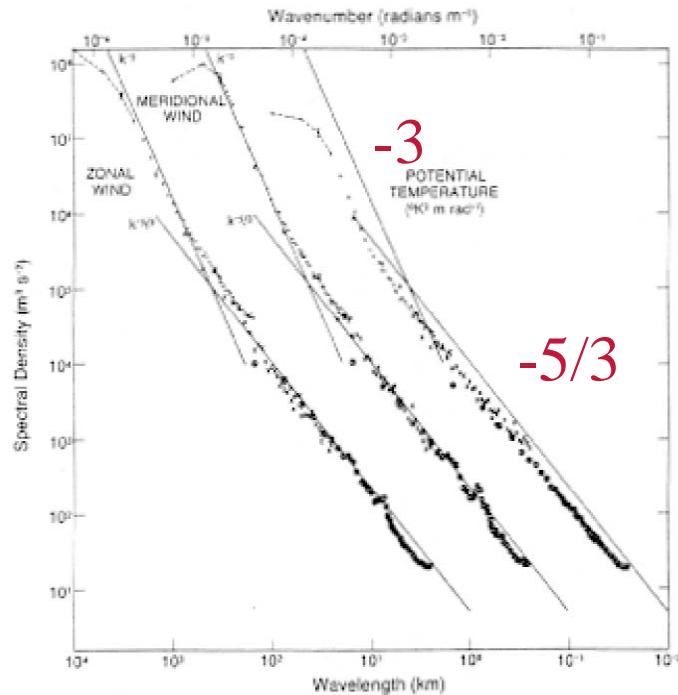
$$\phi_2 = \tilde{\mathbf{u}}(\mathbf{k}) \cdot \mathbf{e}_2(\mathbf{k})$$

$$= \frac{\sqrt{k_x^2 + k_y^2 + k_z^2}}{\sqrt{k_x^2 + k_y^2}} \tilde{w}$$

(wavy)

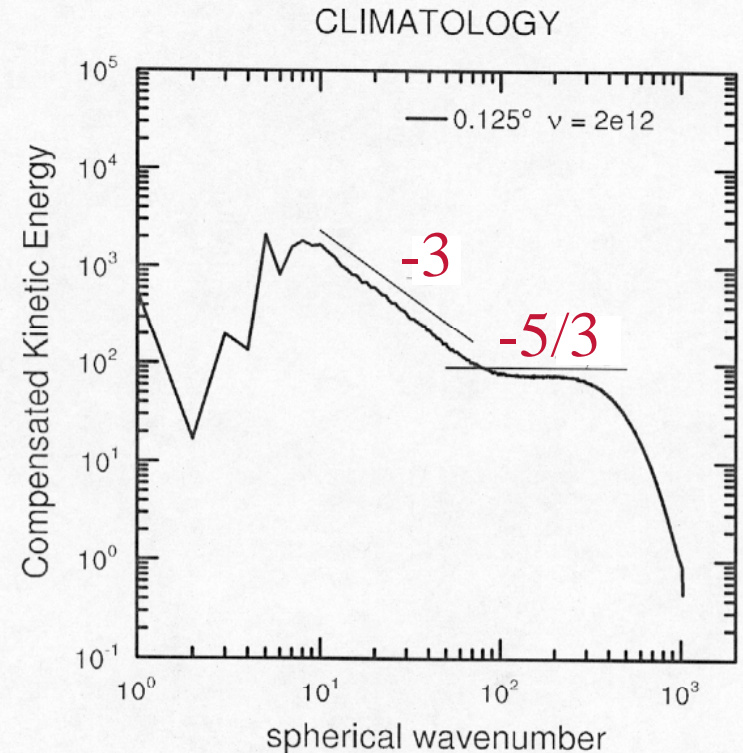
cf: Giga & Yoshida, Babin, Mahalov, Nicolaenko

Transition in Energy Spectrum for Rotating and Stratified turbulence



Nastrom-Gage's atmospheric
observation (1985)
(*JAS* 42 950-960.)

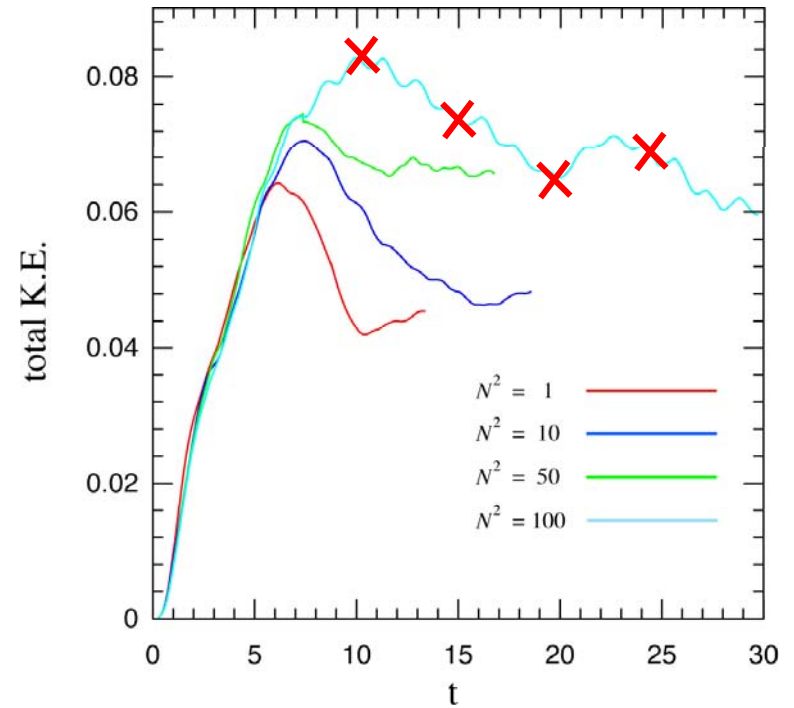
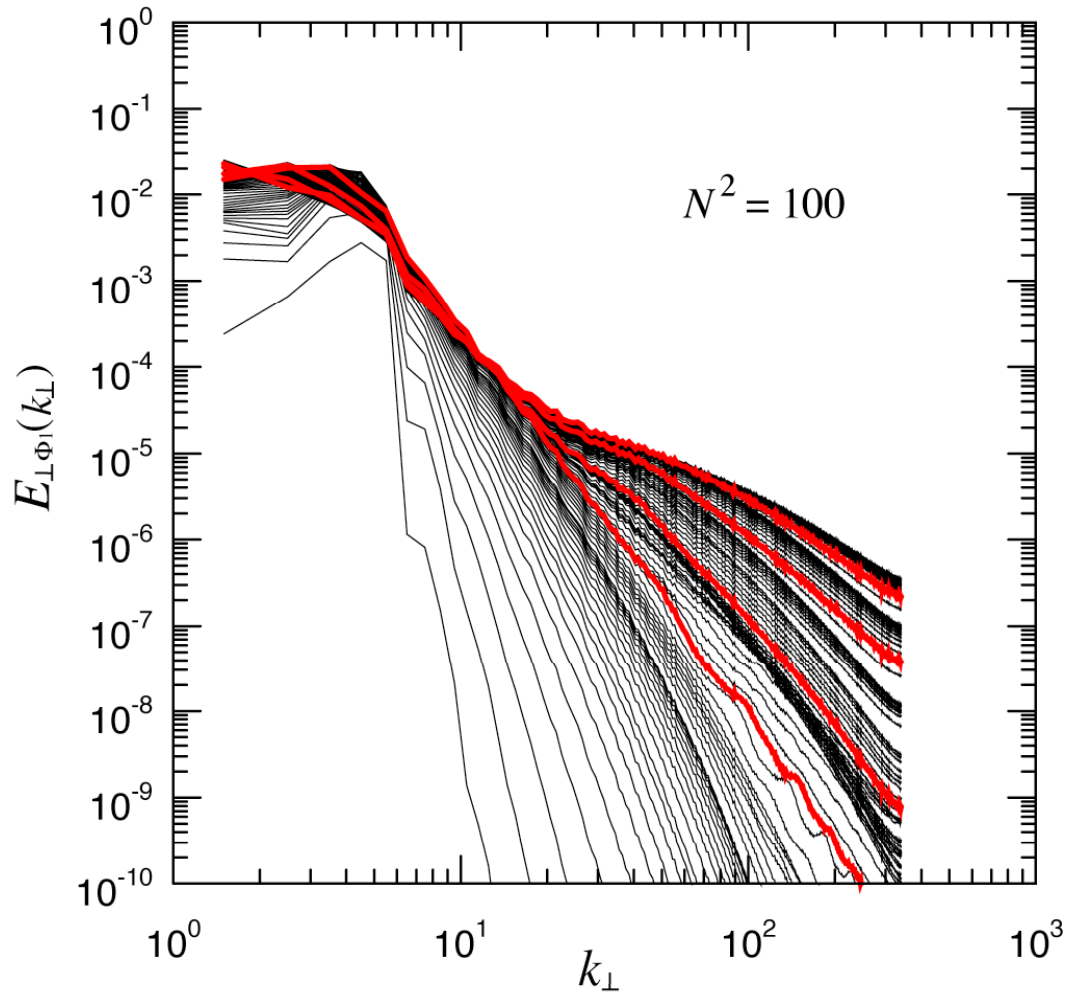
*Both stratification
and rotation are essential*



Mark Taylor's climate model
simulation (2008)
(CCSM project at NCAR)

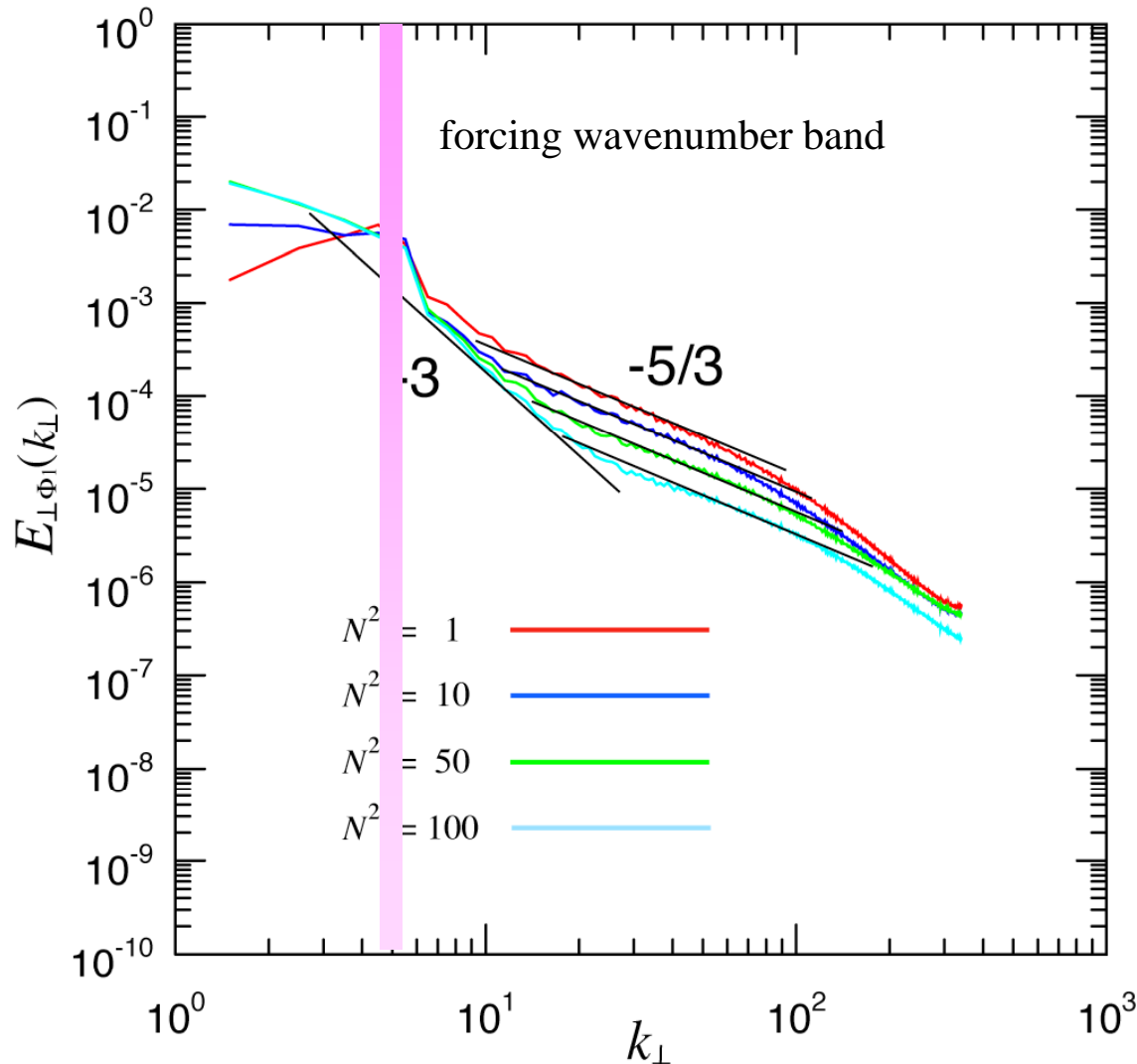
- 3 : enstrophy cascade for Quasi-Geostrophic turbulence ($\sim 2D$)
- 5/3 : Kolmogorov turbulence (3D)

History of Φ_1 energy spectra ($N^2=100$)



First, steep spectrum ($\sim k^{-3}$) develops then small scales rise.

$\Phi_1(k_\perp)$ spectra for various N



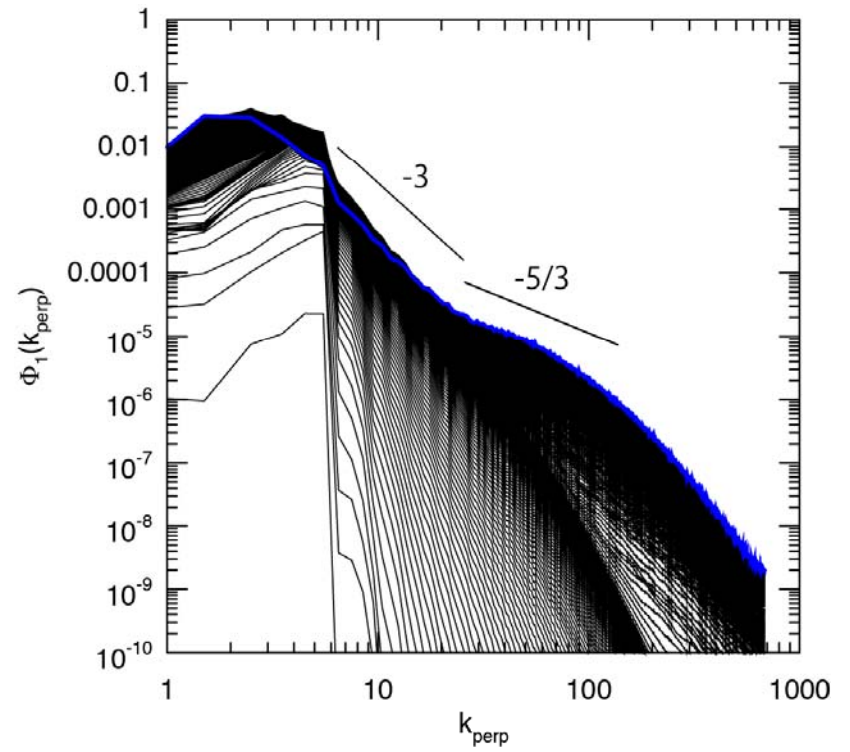
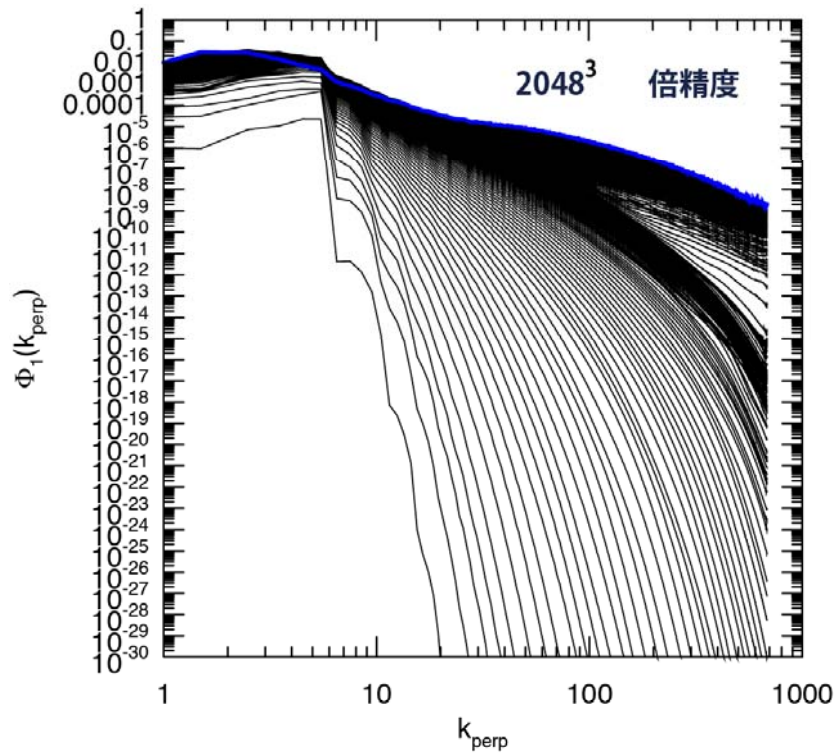
- ◆ $-5/3$ for small scales (different coefficient)
- ◆ -3 for large scales (same coefficient)
- ◆ There's a sharp transition

How to scale them ?

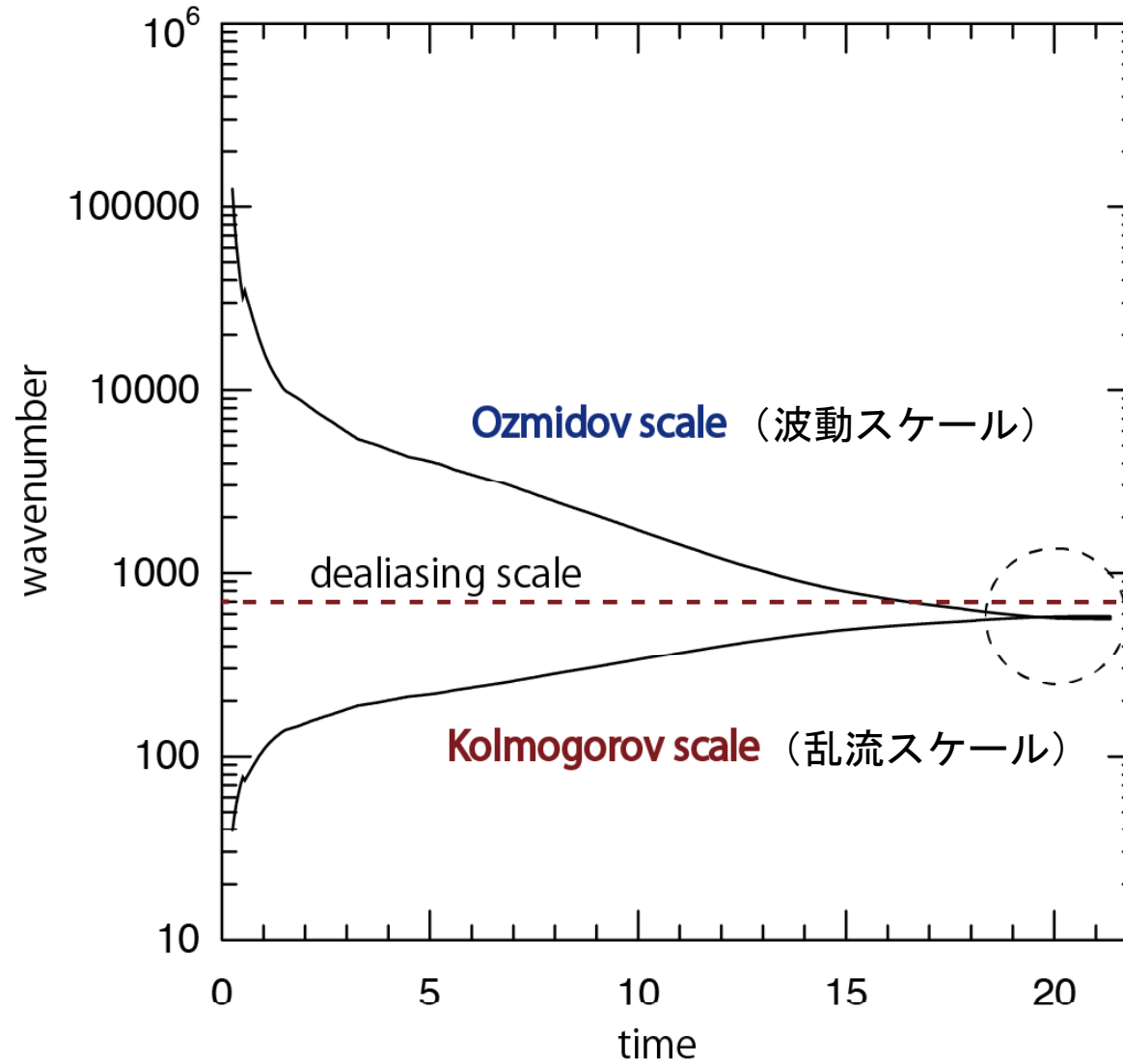
guidelines:

- ◆ The Kolmogorov const. is universal. (Sreenivasan 85)
- ◆ The coefficient for large Scales doesn't depend on N

高解像度／倍精度での検証

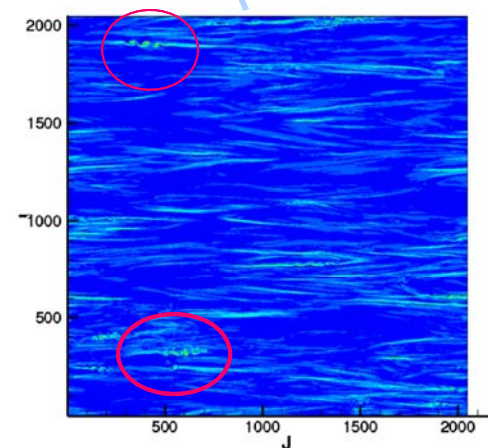
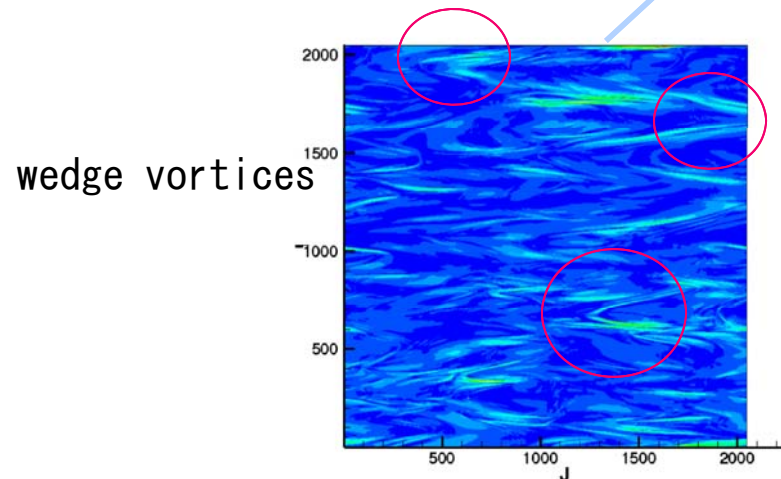
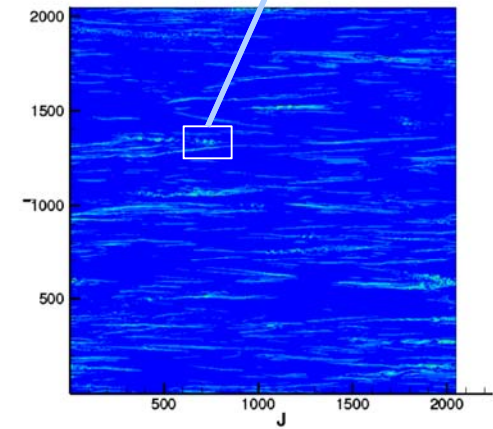
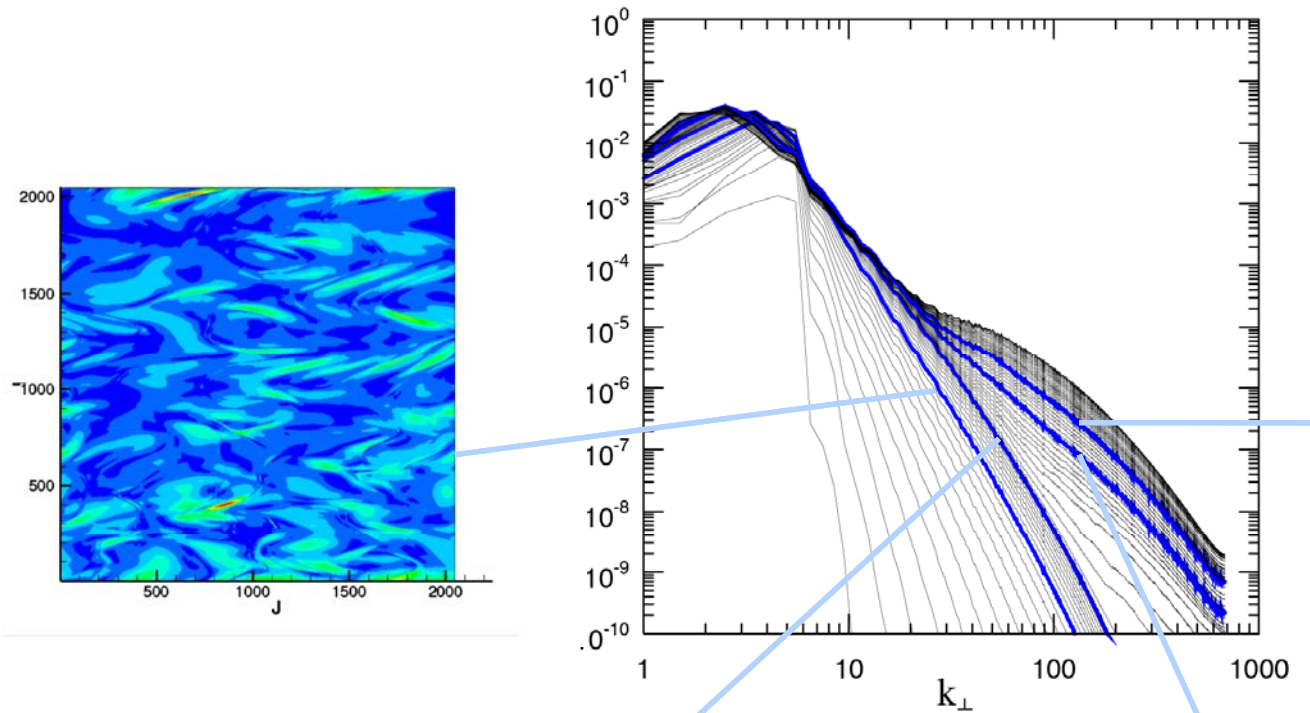
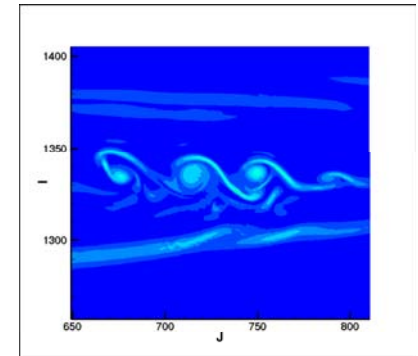


2つのスケールの時間変化



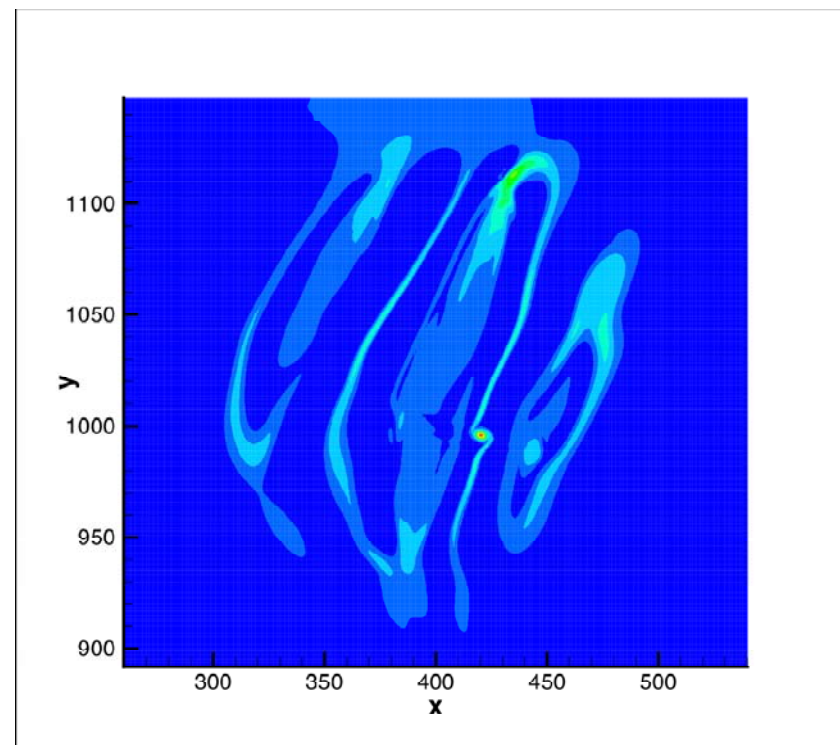
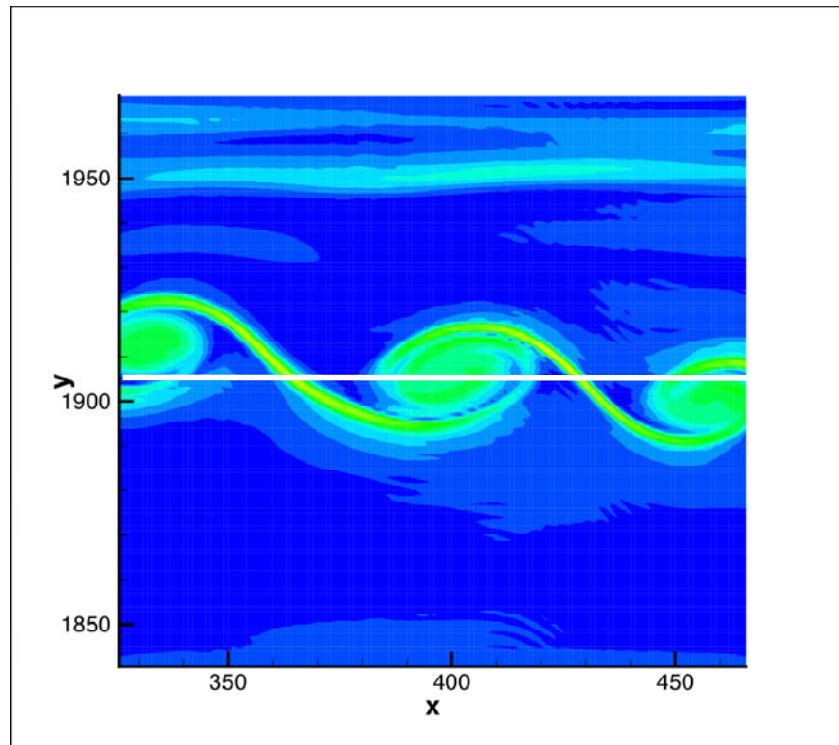
Growth of vortical structures in time

Enstrophy contours in a vertical slice

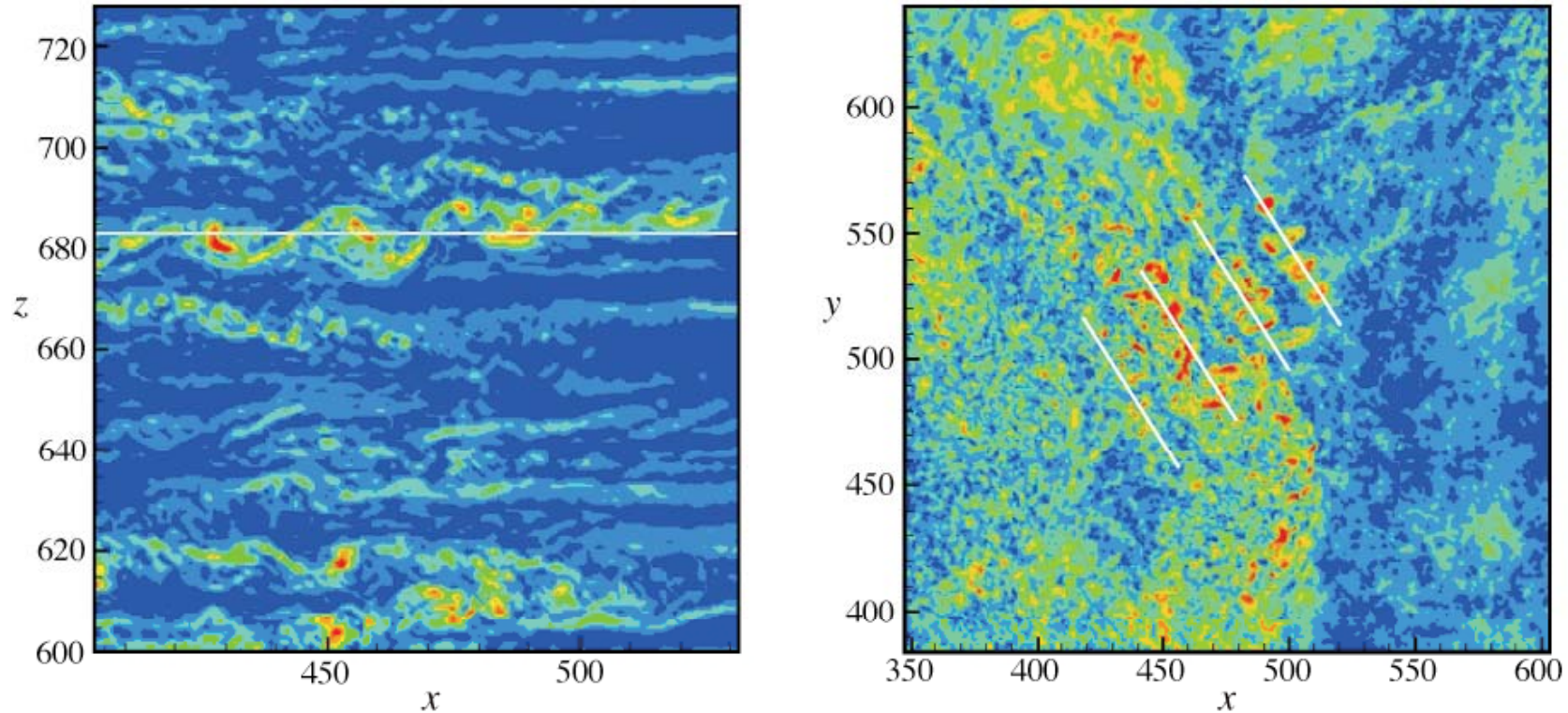


Kelvin-Helmholtz
billows

Kelvin-Helmholtz 渦の構造

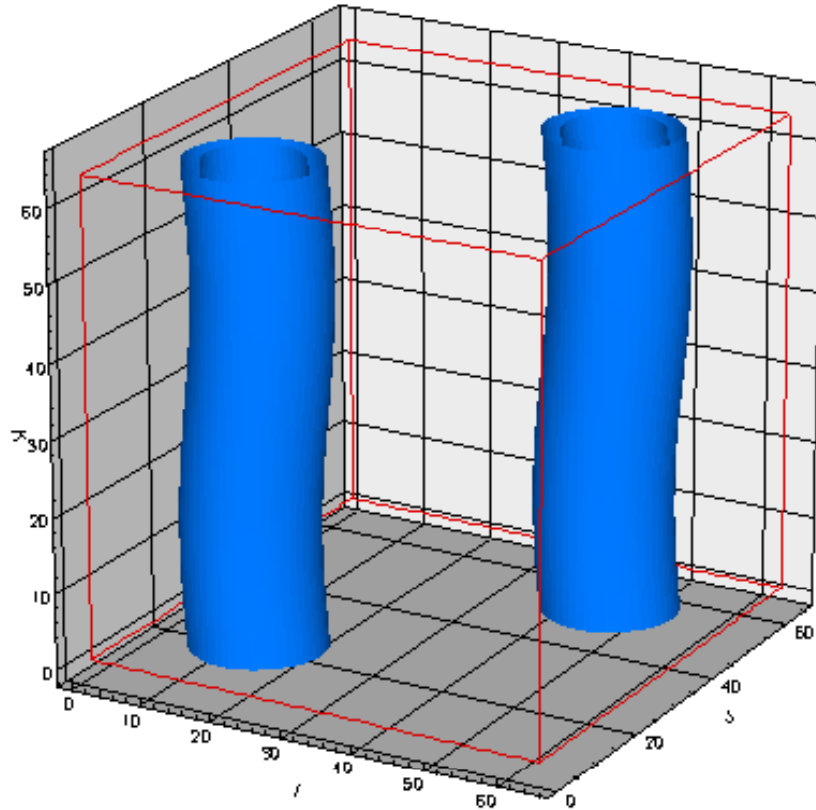


Enstrophy contours (blow-up)



- ◆ Kelvin-Helmholtz billows are observed in the vertical.
- ◆ The billows are not single rollers and chopped in the horizontal.

Wedge構造の生成



強安定成層極限モデル

Riley, Metcalfe, Weissman (1981)



移流拡散方程式（準線形）

Majda, A.J. & Grote M.J.
Model dynamics and vertical
collapse in decaying strongly
stratified flows.

Phys. Fluids **9** (1997) 2932-2940.

まとめ

- ◆ 成層乱流の水平方向のエネルギースペクトルはある波数で2次元から3次元への遷移をおこす。
- ◆ エネルギースペクトルの遷移は乱流中の乱流と波動の相互作用による渦構造の安定性に関係があると思われる。