「名古屋大学HPC計算科学連携研究プロジェクト」
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地球流体乱流の数値解析

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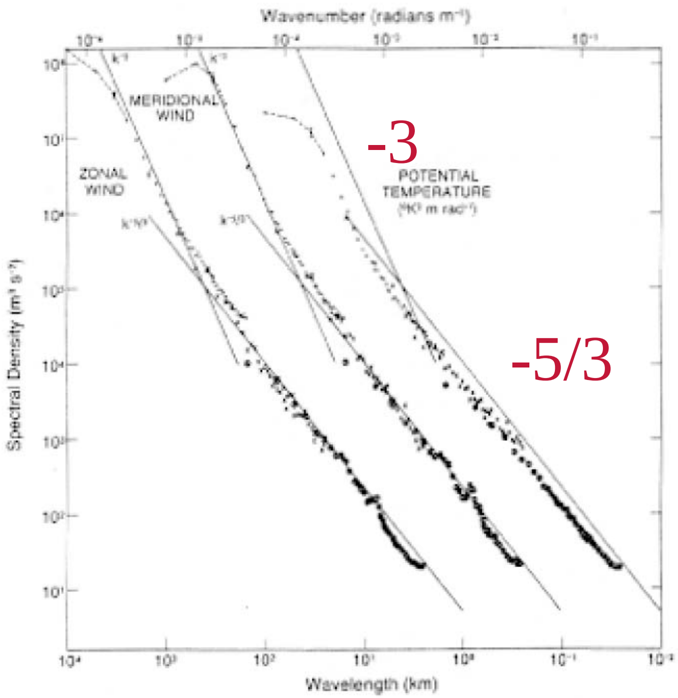
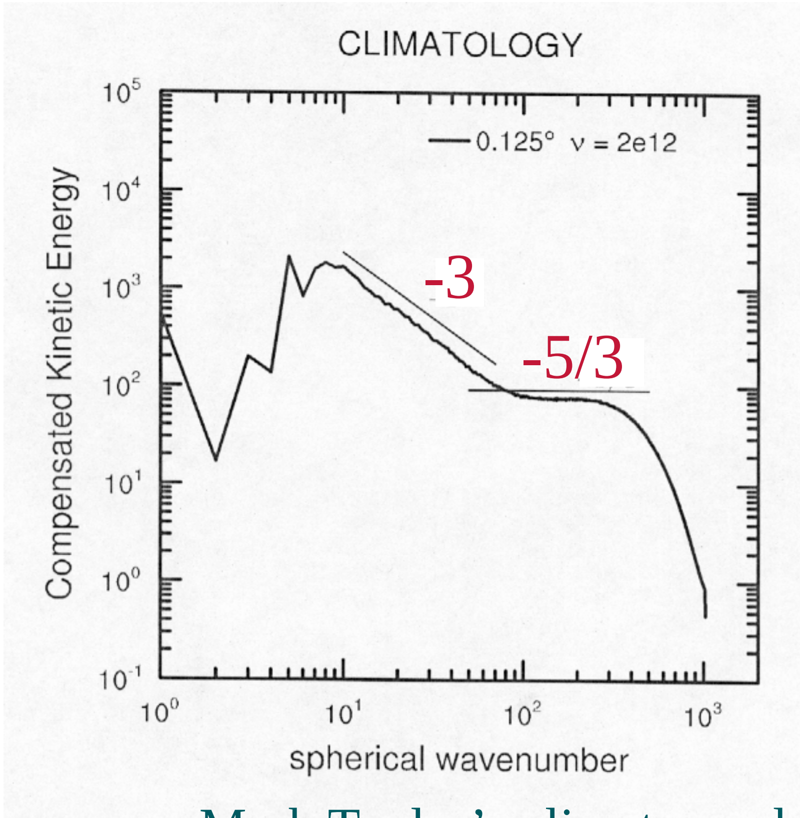
Transition in Energy Spectrum for Rotating and Stratified turbulence



Nastrom-Gage’s atmospheric observation (1985) (*JAS* **42** 950-960.)



Mark Taylor’s climate model simulation (2008) (CCSM project at NCAR)

Both stratification and rotation are essential

-3 : enstrophy cascade for Quasi-Geostrophic turbulence (~2D)

-5/3 : Kolmogorov turbulence (3D)

Transition in Energy Spectrum for Stratified turbulence

Observations: (in the ocean)

$k_z^{-2 \sim -3}$ → $k^{-5/3}$

Garret-Munk spectrum   Kolmogorov spectrum

Munk (1981), Garrett *et.al* (1981)

transition wavenumbe: $k_c \sim \sqrt{N^3/\varepsilon}$ (Ozmidov scale)

Theory: Munk (1981), Garrett *et.al* (1981), Lumley (1964), Holloway (1983)

All support the Ozmidov scale for transition

Simulation: Carnevale, Briscoline & Orlandi (2001)   LES at 128^3

Yoshida, Ishihara & Kaneda (2002)   LES up to 512^3

~ Ozmidov for transition

Waite & Bartello (2004)   DNS + hyperviscosity

(Waite & Bartello (2004) for the review)

Navier-Stokes equation
with the Boussinesq approximation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} + \theta\hat{\mathbf{z}} + \mathbf{F}$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u}\cdot\nabla)\theta = \kappa\nabla^2\theta - N^2 w$$

$$\nabla\cdot\mathbf{u} = 0$$

where

$\mathbf{u} = (u,\ v,\ w)$: velocity

θ : temperature fluctuations

$N^2 = \dfrac{g\alpha}{T_0}\dfrac{\partial \overline{T}}{\partial z}$: Brunt - Väisälä frequency

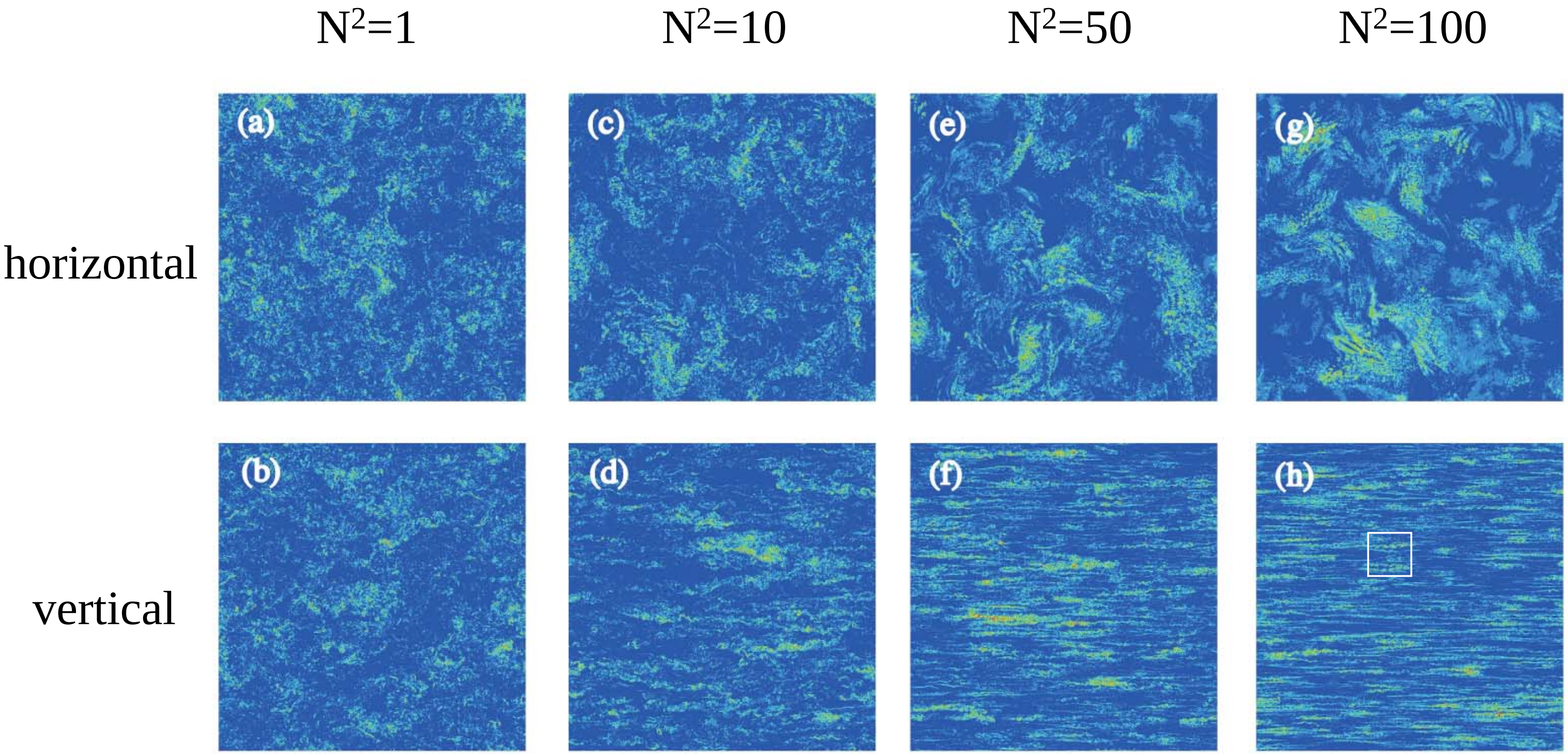
$\mathbf{F}$: Forcing (horizontal)

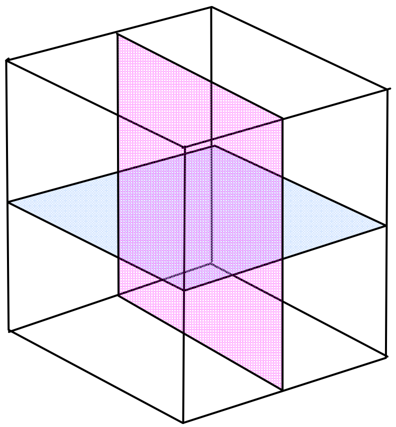
Numerical Methods

- forced simulations
- 2π-periodic box with 1024^3 grid points ($R_\lambda \sim 300$)
- 3rd order time-marching scheme
- Initial energy spectrum : $E(k) = 0$
- Force horizontal velocity components
- Add red noise to modes within a wave number band ($k_f \sim 5$)

  → Solving Ornstein-Uhlenbeck process (2nd order stochastic ODEs)

Enstrophy contours

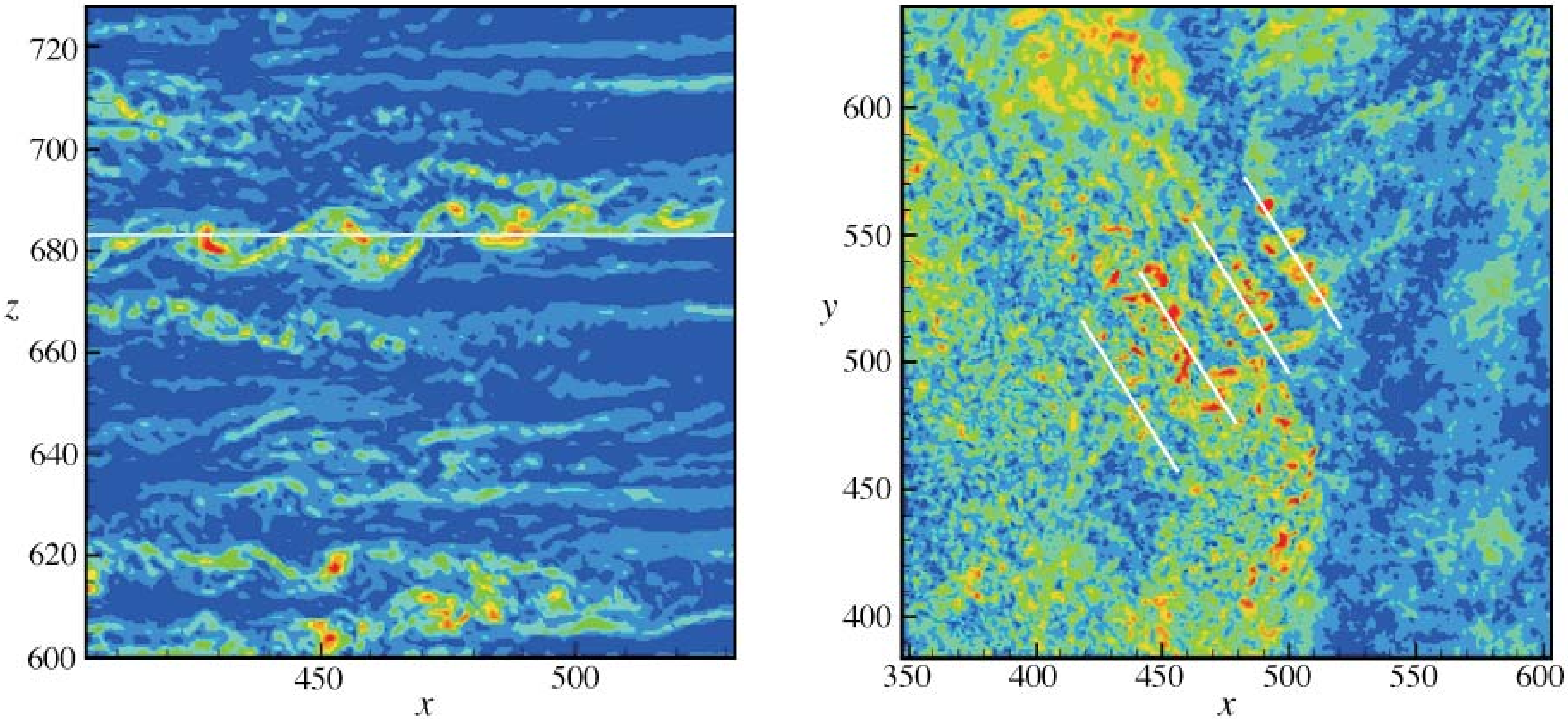




As N^2 becomes large;

- large scale clusters and elongated streaks appear in the horizontal plane.
- thin layers and wedge structures develop in the vertical plane.

Enstrophy contours (blow-up)



- Kelvin-Helmholz billows are observed in the vertical.
- The billows are not single rollers and chopped in the horizontal.

Characteristics of stratified turbulence

- Composite of "waves" and "turbulence"

  ⇒ "*Craya-Herring decomposition*" to separate waves and turbulence

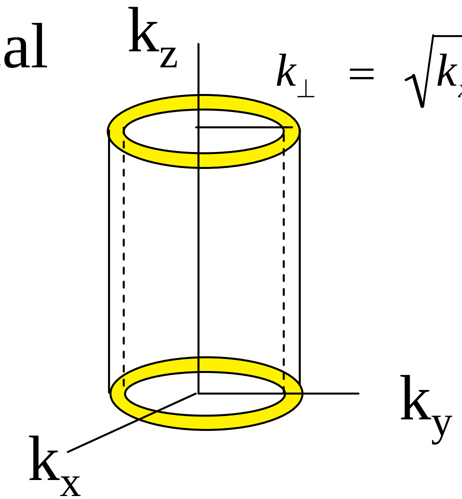
- Highly anisotropic

  ⇒ Need suitable avaraging

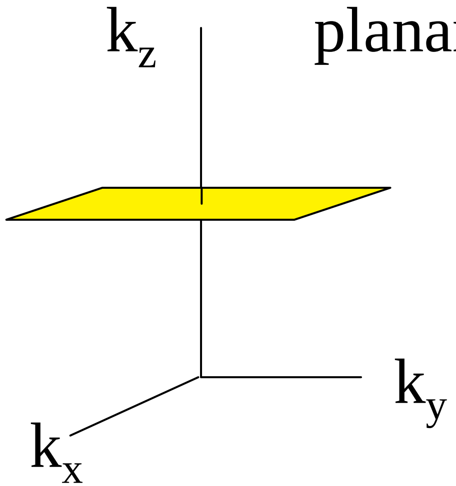
$$\mathbf{k} = k_\perp \otimes k_z$$

cylindrical

$$k_\perp = \sqrt{k_x^2 + k_y^2}$$



planar



"Craya-Herring" decomposition

$$\nabla \cdot \mathbf{u} = 0 \quad \Longleftrightarrow \quad \mathbf{k} \cdot \tilde{\mathbf{u}} = 0$$

incompressibility

$\tilde{\mathbf{u}}$ is spanned by two independent vectors perpendicular to **k**

$$\mathbf{e}_1(\mathbf{k}) = \frac{\mathbf{k} \times \hat{\mathbf{z}}}{\|\mathbf{k} \times \hat{\mathbf{z}}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2}} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}$$

$$\mathbf{e}_2(\mathbf{k}) = \frac{\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{z}}}{\|\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{z}}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2 + k_z^2}\sqrt{k_x^2 + k_y^2}} \begin{pmatrix} k_z k_x \\ k_z k_y \\ -(k_x^2 + k_y^2) \end{pmatrix}$$

$$\mathbf{e}_3(\mathbf{k}) = \frac{\mathbf{k}}{\|\mathbf{k}\|} = \frac{1}{\sqrt{k_x^2 + k_y^2 + k_z^2}} \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}$$

orthnormal coordinates

$$\tilde{\mathbf{u}}(\mathbf{k}) = \phi_1 \mathbf{e}_1(\mathbf{k}) + \phi_2 \mathbf{e}_2(\mathbf{k})$$

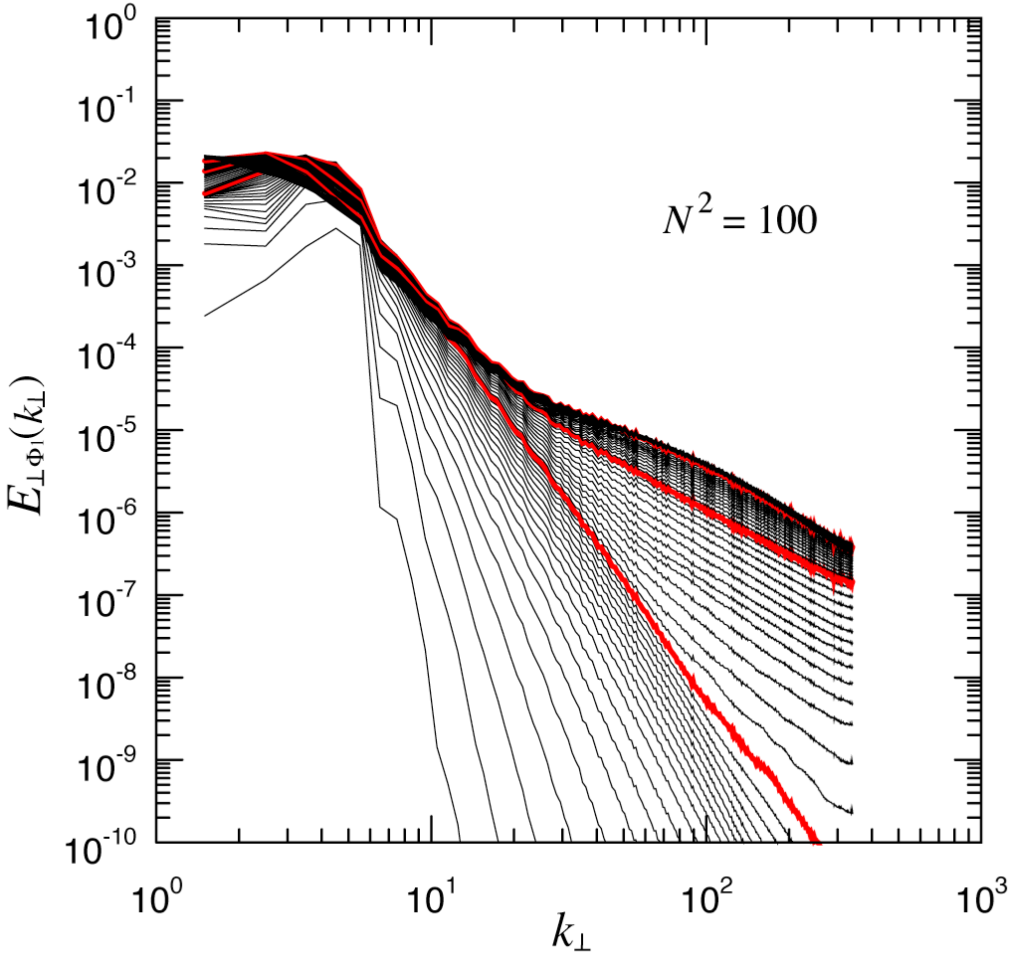
$$\phi_1 = \tilde{\mathbf{u}}(\mathbf{k}) \cdot \mathbf{e}_1(\mathbf{k}) = \frac{1}{\sqrt{k_x^2 + k_y^2}}(k_y \tilde{u} - k_x \tilde{v}) = \frac{i}{\sqrt{k_x^2 + k_y^2}} \tilde{\omega}$$

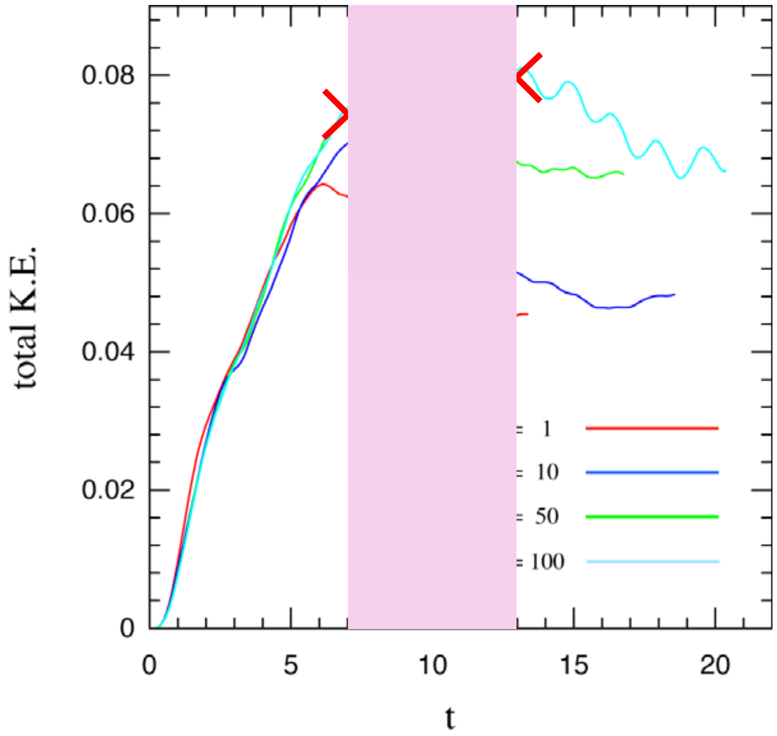
(vortex, rotation)

$$\phi_2 = \tilde{\mathbf{u}}(\mathbf{k}) \cdot \mathbf{e}_2(\mathbf{k}) = \frac{\sqrt{k_x^2 + k_y^2 + k_z^2}}{\sqrt{k_x^2 + k_y^2}} \tilde{w}$$

(wave, divergence)

History of Φ_1 energy spectra (N^2=100)





First, steep spectrum ($\sim k^{-3}$) develops then small scales rise.

History of buoyancy Reynolds number

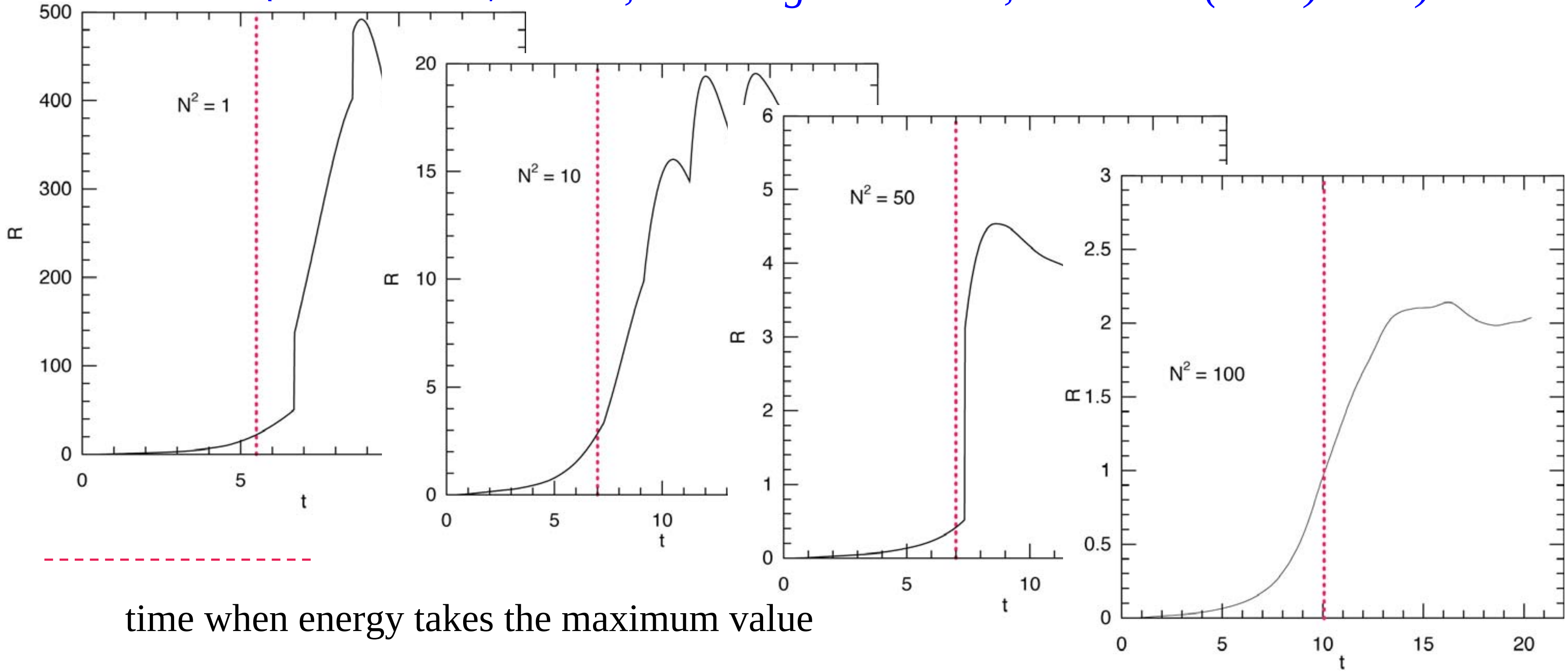
$$R = \mathrm{Fr}_h^2 \mathrm{Re} = \frac{\varepsilon}{\nu N^2}$$

$$\rightarrow \left[\sqrt{\frac{\varepsilon}{\nu \mathrm{N}^2}} \Big/ \left(\nu^3 / \varepsilon \right)^{1/4} \right]^{4/3} = \left[\mathrm{L_O} / \mathrm{L_K} \right]^{4/3}$$

$\mathrm{L_O}$: Ozmidov scale $\quad$ $\mathrm{L_K}$: Kolmogorov scale

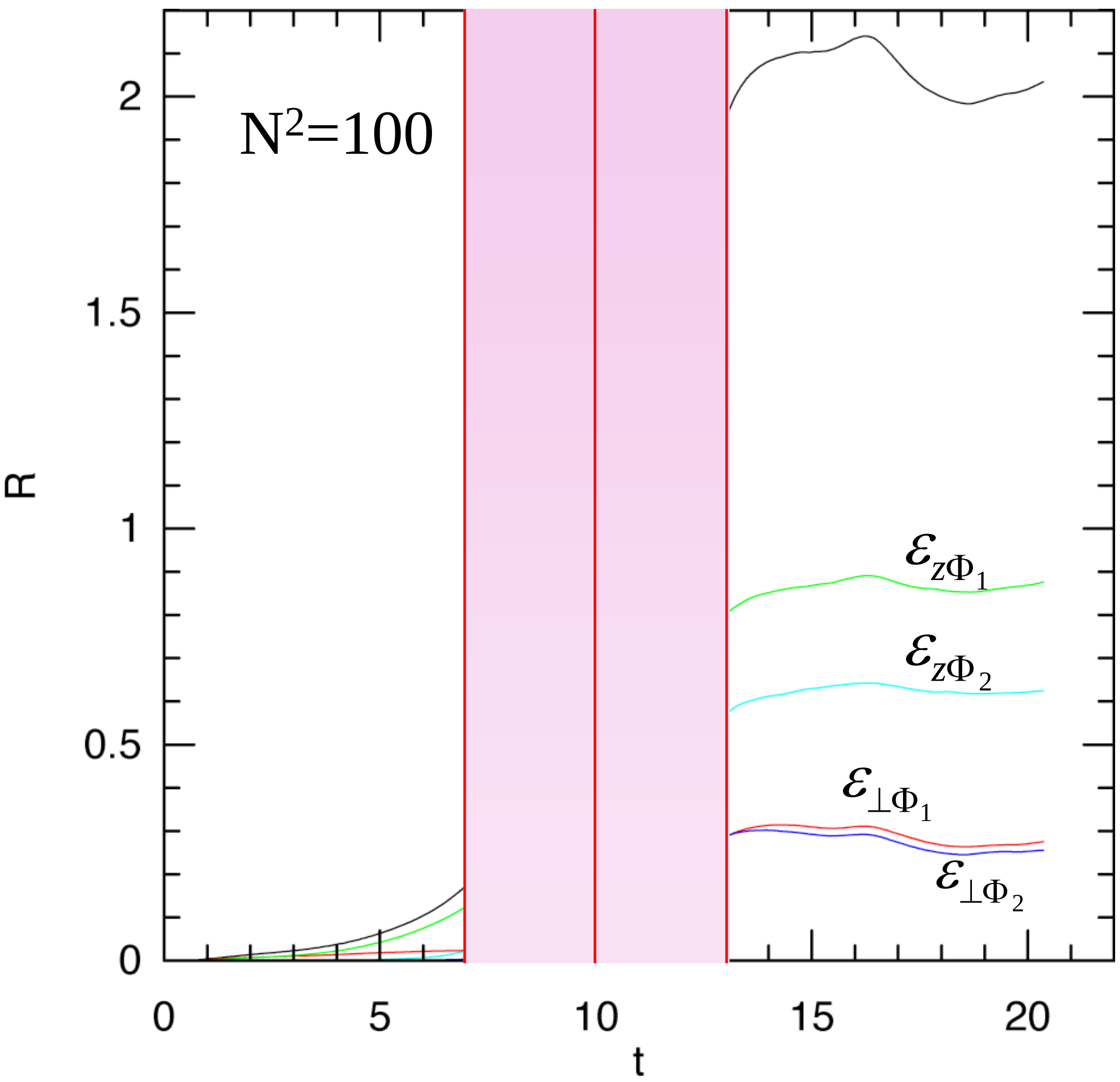
R < 1 : steep spectrum, R > 1: -5/3.

*(Brethouwer, Billant, Lindborg & Chomaz, JFM **585** (2007) 343.)*

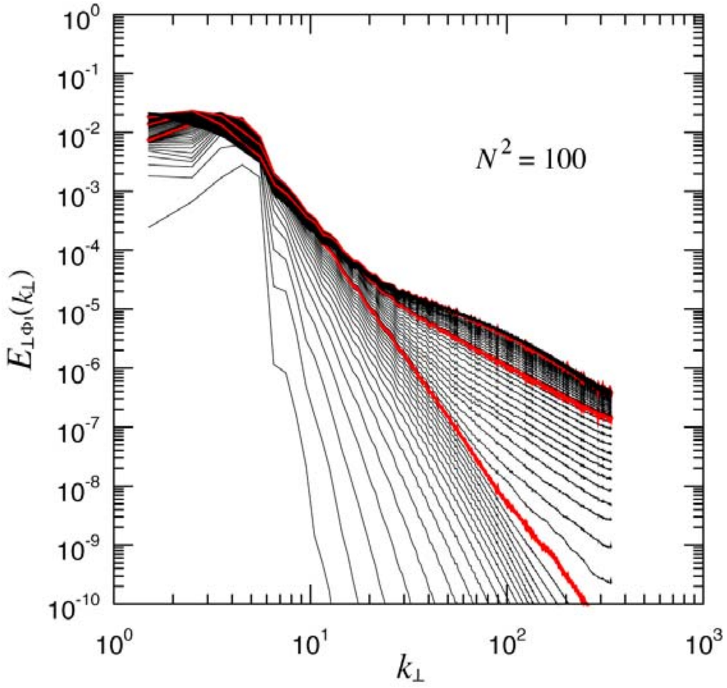


time when energy takes the maximum value

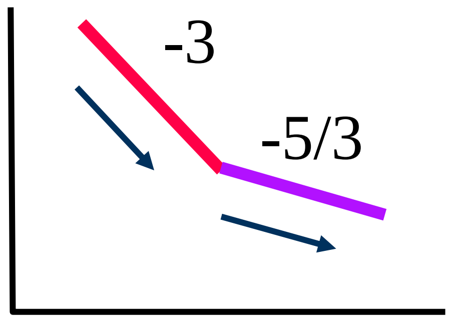
History of buoyancy Reynolds number



$$R = \mathrm{Fr}_h^2\,\mathrm{Re} = \frac{\varepsilon}{\nu N^2}$$



How to understand these observations?

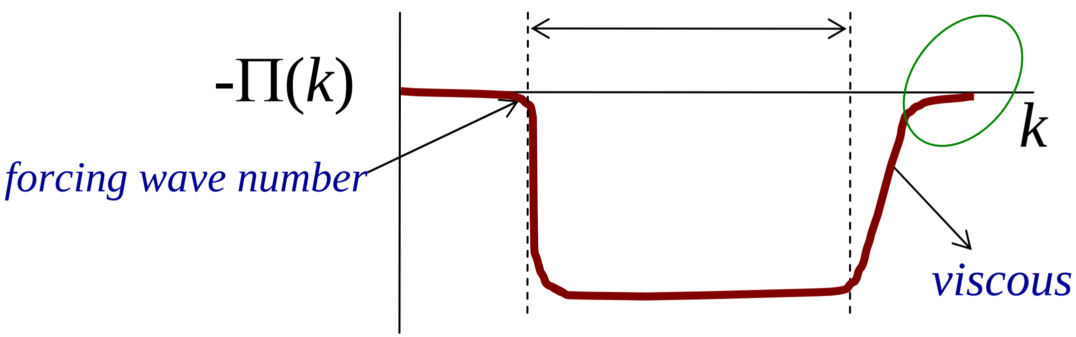


◆ More than one inertial ranges?
◆ How to deal with anisotropy?

< *review* : Kolmogorov (homogeneous isotropic) turbulence >

$$\Pi(k) = -\int_0^k \hat{T}(k)\, dk \qquad \text{(flux function)}$$

spherical average of energy transfer function



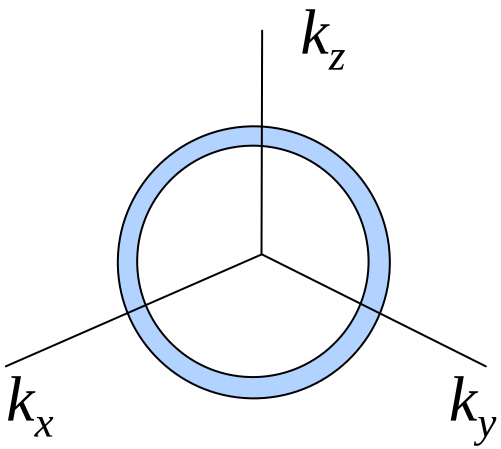
goes back to 0 because of the symmetry of the coefficients in the wave number space.

See for example:
T. Aoyama, T. Ishihara, Y. Kaneda, M. Yokokawa, K. Itakura & A. Uno *J. Phys. Soc. Jpn* **74**(2005) 3202-3212

Flux function

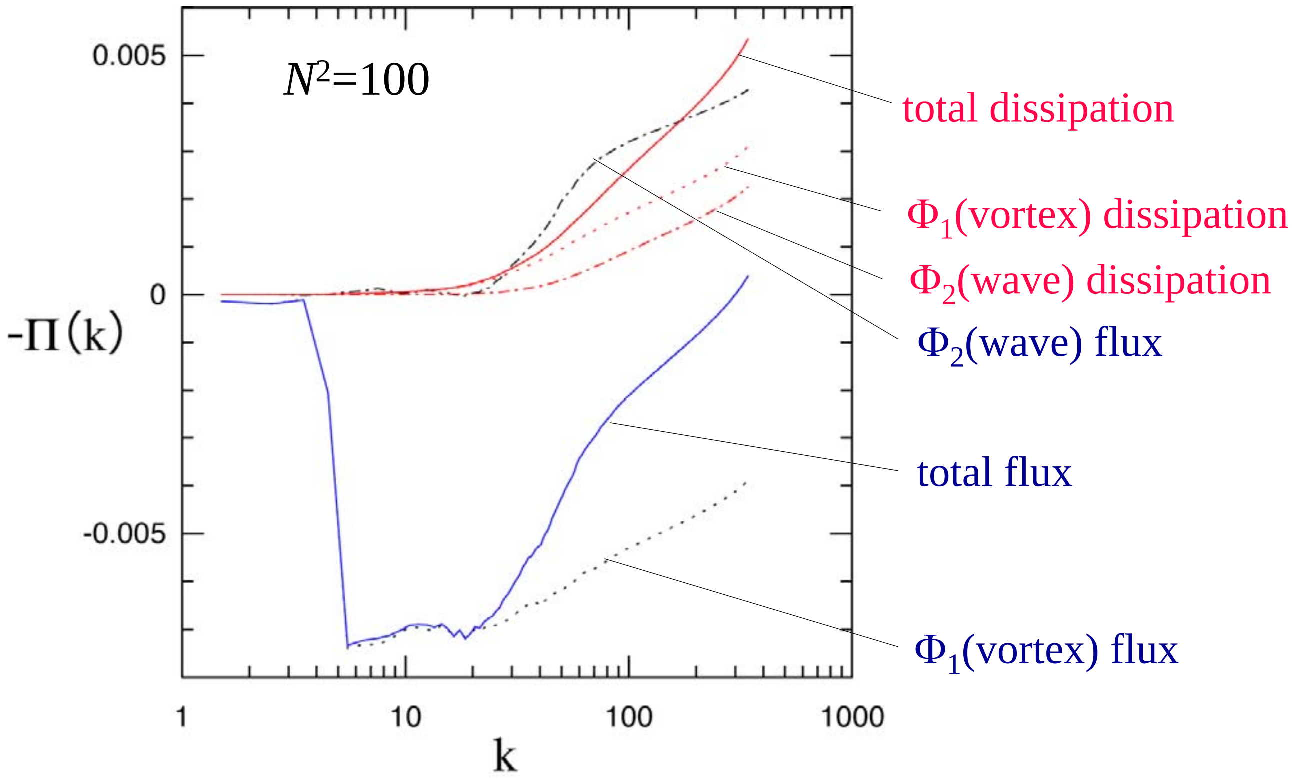
$$\hat{T}(k)\Delta k = \sum_{k-\Delta k/2<|\mathbf{k}|<k+\Delta k/2} T(\mathbf{k})$$ (spherical average)

To check energy conservation!



$$\Pi(k) = -\int_0^k \hat{T}(k)\, dk$$ (flux function)

$$D(k) = \int_0^k 2\nu k^2 \hat{E}(k)\, dk$$ (accumulated dissipation)



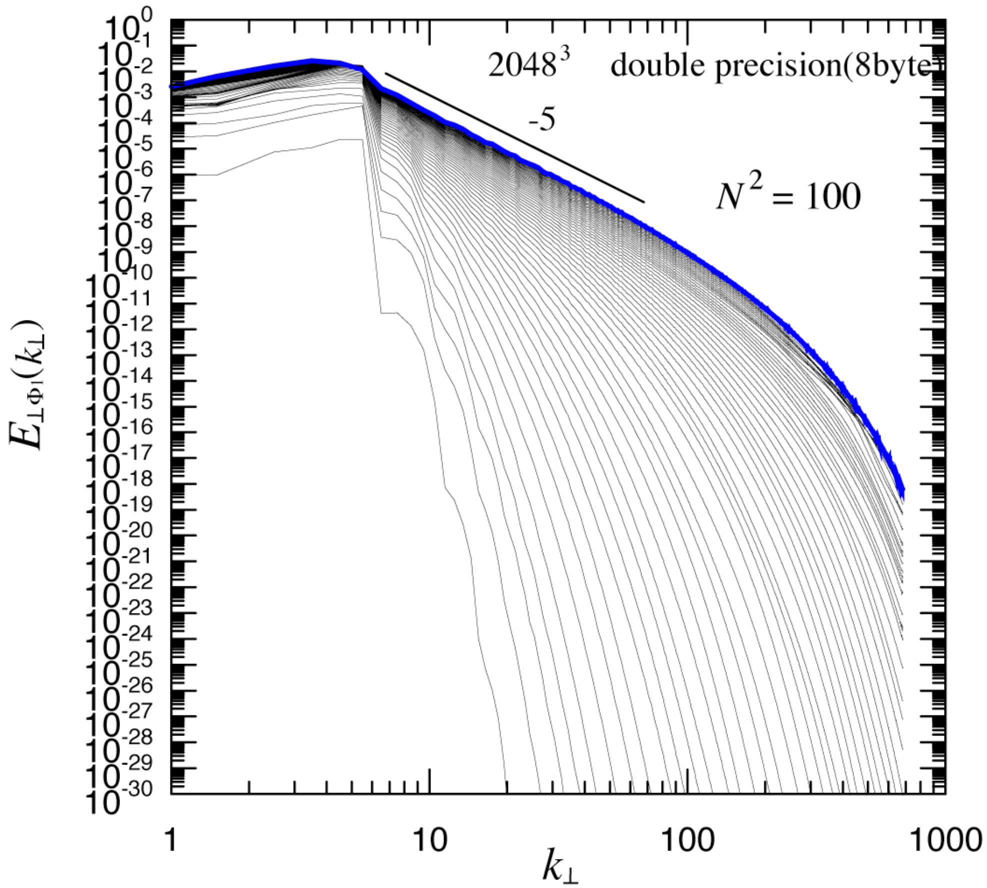
- There is a constant flux region around 5<k<20

  ⟶ inertial range in the sense of Kolmogorov
- Total flux goes back to 0, but not for Φ_1 and Φ_2 fluxes

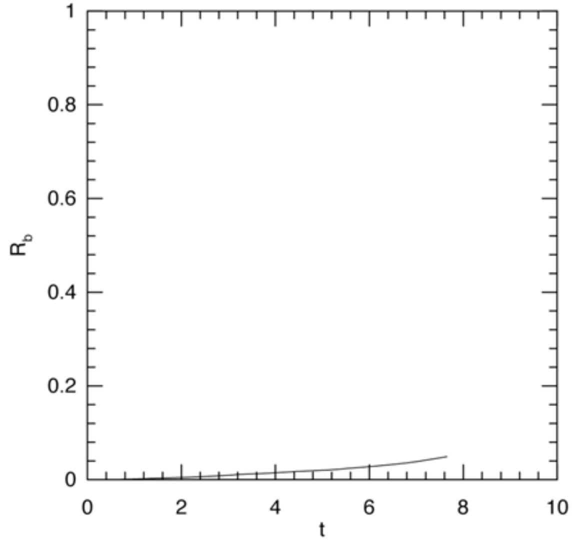
  ⟶ There is total energy conservation, but there is energy exchange between Φ_1 and Φ_2
- Wide range of dissipation

  ⟶ dissipation seems enhanced by the wave flux

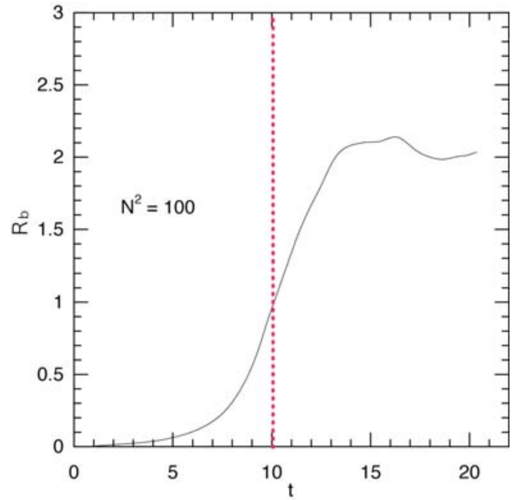
Verification with 2048^3 (double precision)



buoyancy Reynolds number



compare with the 1024^3 run



Summary

- Energy spectra are investigated for stably stratified turbulence with 1024^3 pseudospectal DNS simulations.

- Horizontal spectra show clear transition from 2D to 3D Kolmogorov spectra.

- Horizontal spectra are scaled by anisotropic dissipation of energy and enstrophy.

- Vertical spectra show a flat part at large scales and tend to have steeper spectrum(-3) as N becomes large.